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Comment on "Resolution of Runge-Kutta-Nystrom Condition Equations through Eighth Order"

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AT the time of publication of the subject paper,[1] the author was obliged to resort to a numerical experiment to eliminate two of the condition equations in deriving the general eighth-order, eight-stage algorithms. Since then the author has succeeded in developing an analytic argument which is, of course, far more satisfying and is now presented in this Comment.

As in the paper, we impose the constraints

$$b_1 = 0, \quad p_7 = 1$$

Then we form a set of five equations by adding Eqs. (α^i) and (α^{i+2}) and subtracting two times Eq. (α^{i+1}) for $i = 1,\ldots, 5$. When these are compared to Eqs. (γ^0), (ϵ^0), (ι^0), (σ^0), (e_2), and we recall that $H_1^0 = 0$, it follows that

$$H_j^0 = \tfrac{1}{2}\, p_j (1-p_j)^2 b_j \quad (j = 2,\ldots,6)$$

Equation (e_5) can now be written as

$$(1-p_2)^2 A_2^4 b_2 + \ldots + (1-p_6)^2 A_6^4 b_6 = 2/8! \qquad (e_5)$$

which is identical to the equation formed by adding Eqs. (ι^0) and (ι^2) and subtracting two times Eq. (ι^1). Therefore, Eq. (e_5) may be discarded if the three equations (ι) are satisfied by the Nystrom parameters. The same operations applied to Eqs. (λ^0), (λ^1), and (λ^2) show that Eq. (e_6) may also be discarded.

All minimal-stage Runge-Kutta-Nystrom algorithms through eighth order are now complete with the condition equations fully resolved analytically.

Finally, we note a typographical error in the paper on page 1016. Two sets of parameters p_0, p_1, p_2, p_3 were cited which Nystrom used to develop fifth-order, four-stage algorithms. The first set should have been

$$p_0 = 0, \quad p_1 = \frac{2}{5}, \quad p_2 = \frac{2}{3}, \quad p_3 = \frac{4}{5}$$

The value for p_3 was omitted in the typesetting.

Reference

[1]Battin, R. H., "Resolution of Runge-Kutta-Nystrom Condition Equations through Eighth Order," *AIAA Journal*, Vol. 14, Aug. 1976, pp. 1012-1021.

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Comment on "Stress Concentration in the Plastic Range"

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THE authors' rationale for using a semigraphical method for determining the plastic stress concentration factor, k_p, from the Neuber equation used with the Ramberg-Osgood stress-strain law, is the claim that the solution of the resulting equation requires a lengthy trial and error procedure.[1] One purpose of this Comment is to show that the procedure need not be "lengthy" when the equation is expressed in nondimensional form. The time required for hand computation with a desk or pocket electronic calculator is directly related to the desired precision of the solution value. A simple root-finder routine has been used in our laboratory for machine computation as well. Another purpose of this Comment is to identify a limitation in calculating K_p which is a consequence of the limitation on the applicability of the Ramberg-Osgood equation and to refer to some earlier experimental results which were compared with the predictions of the Neuber theory.

Using the same notation as Ref. 1, the Neuber equation can be written as

$$K_{el}^2 = \sigma\epsilon / p\epsilon_p \qquad (1)$$

It is assumed that the reference stress, p, is a known quantity. By suitable algebraic manipulation, the Ramberg-Osgood approximation of the stress strain curve can be transformed to

$$\sigma\epsilon = (\sigma_y^2/E)\,[(\sigma/\sigma_y)^2 + (3/7)(\sigma/\sigma_y)^{m+1}] \qquad (2)$$

and similarly

$$p\epsilon_p = (\sigma_y{}^2/E)\,[(p/\sigma_y)^2 + (3/7)(p/\sigma_y)^{m+1}] \qquad (3)$$

Substitution of Eqs. (2) and (3) into Eq. (1) results in an equation of the form

$$Z^2 + (3/7)Z^{m+1} - B = 0 \qquad (4)$$

Where $Z = (\sigma/\sigma_y)$ and

$$B = K_{el}^2 [(p/\sigma_y)^2 + (3/7)(p/\sigma_y)^{m+1}] \qquad (5)$$

The second term in the brackets in Eq. (5) is vanishingly small when p is elastic.

Numerical solutions, accurate to one part in a thousand, can be found in times comparable to that required for the procedure of Ref. 1 on a pocket electronic calculator. For machine computation, I have used a simple root-finder routine as part of a more extensive program for computation

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